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The Comparison of Finite Element and Finite Difference Methods in Buckling Analysis of Plate Bending

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Abstract

Plate bending is commonly used nowadays, such as water reservoirs, tanks, domes, concrete dams and etc. Therefore, appropriate methods should be used to analyze these structures. In some cases, however, these analytical solutions are not always possible, and we have to search for numerical solutions. The use of numerical approaches enables the engineer to expand his or her ability to solve design problems of practical significance, The governing equation of thin plate based on classical plate theory (CPT) along with finite element method (FEM) and finite difference method (FDM) are used to solve the equations . The model in ANSYS get buckling local of the plate; besides, to validate the procedures and verify whether it is in good accordance with those methods. At least, the efficiency is computational cost of the procedures compared with each other. The plate model is adopted for the structures. The material, plates of all models, is subject to Hooke’s law and homogeneous. The structures are assumed to be simply supported at the ends. The study is based on the numerical methods which are compared with the exact method. The nonlinear equations of stability are solved with finite element and finite different methods and compared with each other.

Key words: Finite element, Finite difference, Buckling, plate

1 Introduction

While the lateral compressive load has not reached the critical state, the plates are resistant and no disorder will occur in them. But with increasing this load, they become deformed. This state is called buckling and this load is called load buckling.

During years, some of various combinations from the different conditions of loading have been evaluated. Stowell et al. [1] were the pioneers of this task. They evaluated the various conditions of loading. Until now, many researches about the buckling of mono axial have been performed in both theoretical and numerical directions. The canonical exact solutions for elastic bending, buckling and free vibration of plates resting on two-parameter foundations were obtained by Lam *et al*. [2] by using Green’s functions. Penga*et al*. [3] presented a mesh-free Galerkin method for free vibration and stability analysis of stiffened Mindlin plates. Rao *et al*. [4] analyzed the stability of moderate to thick rectangular plates by using a triangular finite element.

There are fewer researches about biaxial buckling. In the represented relations of the various sources, some complicated theories have been used.

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On this paper, the biaxial buckling loads of plate have been analyzed. At first, this buckling has been analyzed by the numerical methods of finite difference and finite element. The rate of preciseness and error has been evaluated by the relations of Timoshenko classic theory. All relations which have been in the BV [5] and GL [6] provisions have been in conformity with the obtained results. Basically, all relations which are in the provisions and the classification of institutions are accordance to the liner estimation of buckling. With regard to the possibility of occurring complex states in the reality, these relations are used with high safety factors. This fact prevents the optimum design of structure.

2 Methods in Use

2.1 Finite difference method

In the finite difference method, the equivalent finite difference equation and the boundary conditions of nodes are used instead of using the differential equation of the plate. The differential equation of plate indicated by Equation 1, is written as Equation 2 with regard to this fact that buckling load is in the direction of X and Y. It must be noted that the shear effect has been ignored.

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y} \qquad (1)$$

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{N}{D}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \qquad (2)$$

The simple equation of finite difference (3) is obtained by replacing Equation 2 with different factors for partial derivations. This buckling load N is calculated by the simultaneous solution for nodes and the boundary conditions.

$$\begin{aligned}&20w_{j,k}-8\left(w_{j+h,k}+w_{j-h,k}+w_{j,k+h}+w_{j,k-h}\right)\\&+2\left(w_{j+h,k+h}+w_{j+h,k-h}+w_{j-h,k+h}+w_{j-h,k-h}\right)\\&+\left(w_{j+2h,k}+w_{j,k-2h}+w_{j-2h,k}+w_{j,k+2h}\right)\\&+\frac{Nh^2}{D}\left(w_{j+h,k}+w_{j-h,k}+w_{j,k+h}+w_{j,k-h}-4w_{j,k}\right)=0\end{aligned}\qquad(3)$$

In the above relations, *h* is the length of quadrate grid, N is the lateral critical load and W is displacement of the plate. In these relations, the plate is divided into grids of four for approximation of first degree n=1, grids of nine for approximation of the second degree n=2, grids of sixteen for approximation of the third degree n=3 and grids of twenty five for approximation of the fourth degree n=4. The dimensions of the grid of sixteen have been shown in Figure 1.

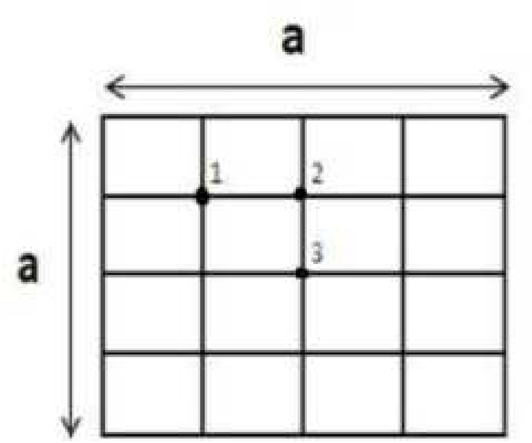

Figure 1: Numbering of 16 grids

2.2 Finite element method

With increasing usage of computers, the method of finite element has been developed. The rate of stress and displacement is calculated by this method in every point of the plate. With using this method, the plate is divided into some finite elements. The equations are written for every element and with regard to the degree of freedom, and then are substituted in the structure system. Ultimately, with considering the boundary conditions and removing some of equations, the critical load of plate is obtained by letting the matrix determinant equal to zero. On this paper, shell 93 element (shown in figure 2) and the sign contract (shown in figure 3) have been used.

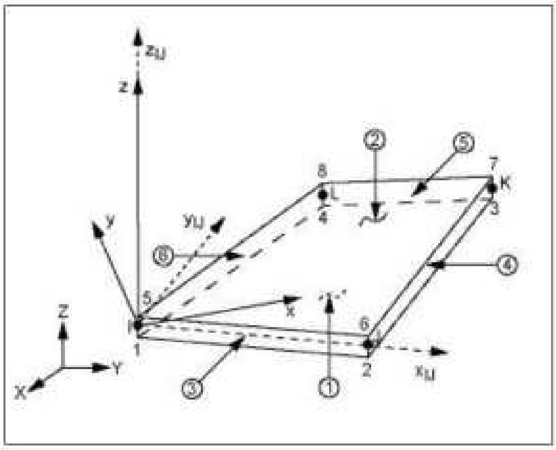

Figure 2: Shell 93 element [6]

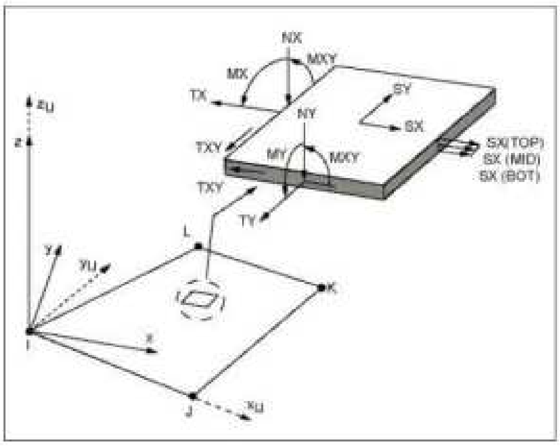

Figure 3: sign contract in Shell 93 element [7]

3 Results of Numerical Methods

3.1 Finite difference method

First, the solution of buckling finite difference for one quadrangular plate on the simple supporter has been considered, while the plate was under the effect of lateral compressive N in two directions of X, Y. As it was mentioned before, the plate is divided into grids of four, nine, sixteen and twenty five, then for every state, the load of buckling (Ncr) is calculated for the first mode of buckling. According to Figure 4 and noting the symmetry, the analysis of one fourth of the surface is sufficient.

We should form independent equations for number of nodes by use of boundary conditions and analyzing one eighth of the plate surface. By putting matrix of coefficient equal to zero we can calculate the load buckling of N for different grids. The results are given in Table (1).

According to Table 1, the rate of accuracy for the first approximation (which includes four grids) has 20% error in comparison to the Timoshenko [8] theory. This error decreases as a result of decreasing the size of grid. With increasing the number of grids (shown in Figures 5 and 6) respectively, the rate of converging to the solution and the rate of error increases.

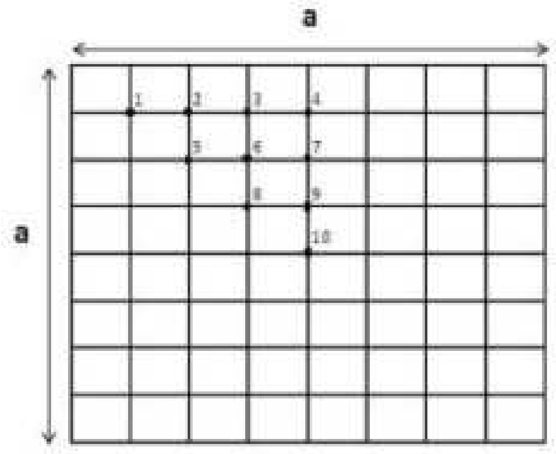

Figure 4: grids of plate in finite difference method

Table 1: results of finite difference method

Error percent	N_{cr}	Number of grids	h	a
20	$16.000D/a^2$	4	a/2	2
10	$18.000D/a^2$	9	a/3	3
4.6	$20.640D/a^2$	16	a/4	4
3.3	$19.098D/a^2$	25	a/5	5

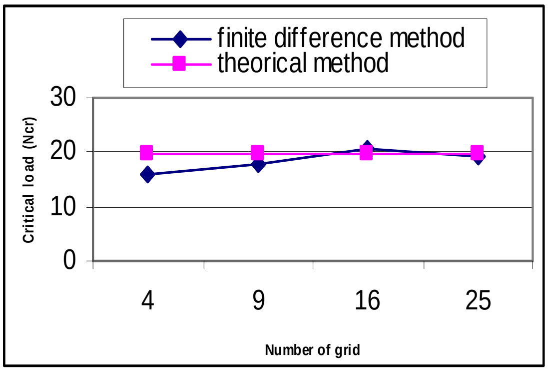

Figure 5: Critical load for increasing of plate grids

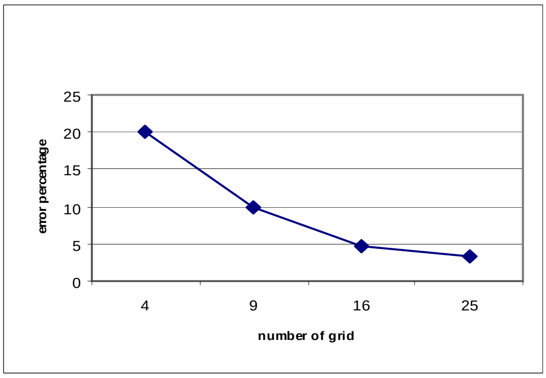

Figure 6: Error changes for increasing of plate grids

3.2 Finite element method

As we mentioned before, in the finite element method, the shell 93 element was used. In this method, we divided the plate into the elements of four, nine… then we analyzed the obtained results for the critical load. This analysis is performed for parameters mentioned below.

E=218.4 GPa modulus of elasticity
μ=0.3 Poisson's ratio
h =0.001(m) thickness of plate
a=0.1 (m) length of plate

Figures 7 and 8 show the results for this method.

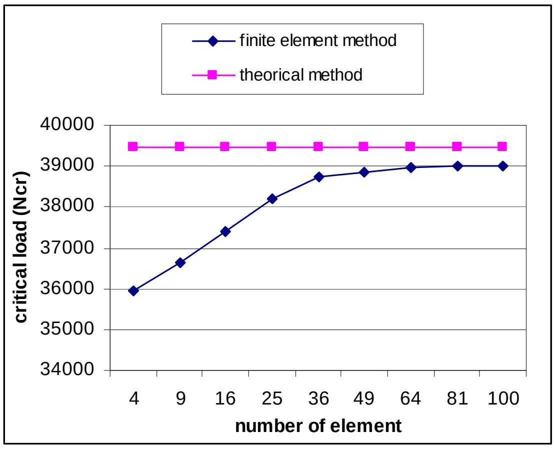

Figure 7: Critical load of plate for different number of elements

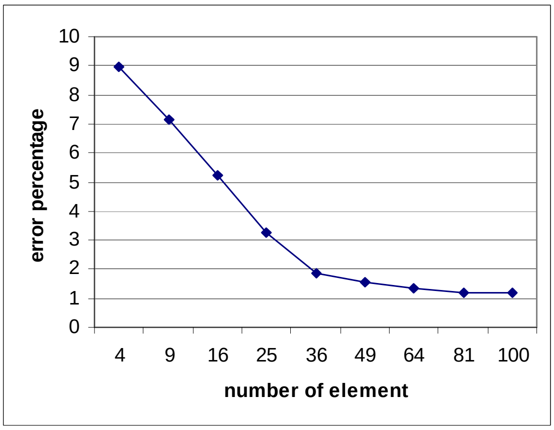

Figure 8: Error changes for different plate elements

The manner of plate deformation has been modeled in the ANSYS software shown in Figures 9 and 10.

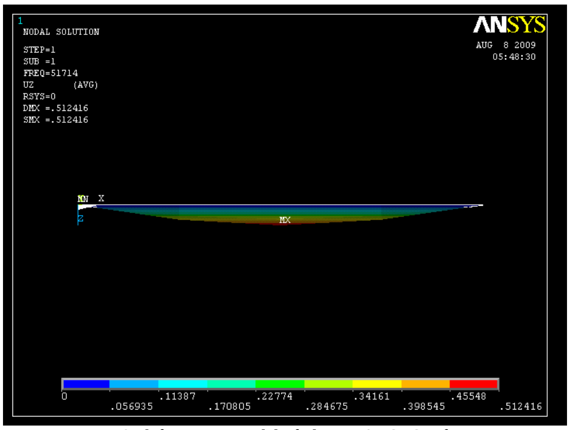

Figure 9: deformation model of plate in ANSYS software

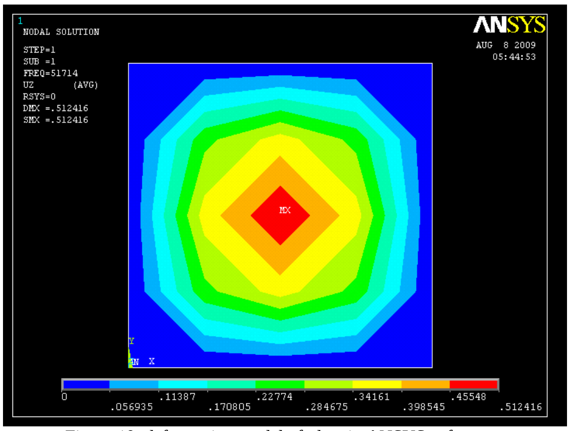

Figure 10: deformation model of plate in ANSYS software

4 Conclusion

As a result of comparing the evaluated methods, we conclude that in the finite element method, the obtained results will approach the correct answer monotonically (descending or ascending) but in the finite difference method, all results will change with one small perturbation. It must be mentioned that the rate of average error in the finite element method is significantly smaller than the finite difference method. There is no any boundary change in the limited element with the increase in the number of element and finally, got the right result. In the beginning of this study the answer achieved and calculate in the node point for the deference between limited element way and limited difference. The Maximum boundary occurred in the plate center. In the method of limited difference, the presence of node in that point is led to accurate answer. While in the method of limited element way, instead of 9 elements there is no node in the center to evaluate. And the problem has solved with the division of the number of elements.

Also, the comparison of numerical and theoretical results with provisional relations indicates that all relations have been provided on the basis of the first mode of plate buckling. With comparing all results which have been obtained by GL and BV code, we concluded that the relations of BV institution have been designed more conservatively. Meanwhile, from the application point of view, the use of presented relation in BV is more convenient.

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